

# learn network inspire

Game Developers'  
Conference

08



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San Francisco

[www.gdconf.com](http://www.gdconf.com)



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# Advanced Soft Shadow Mapping Techniques

Louis Bavoil  
NVIDIA Developer Technology



# Why Soft Shadows?

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- ⊕ Antialiasing

Filtering the shadow edges

- ⊕ Area lights

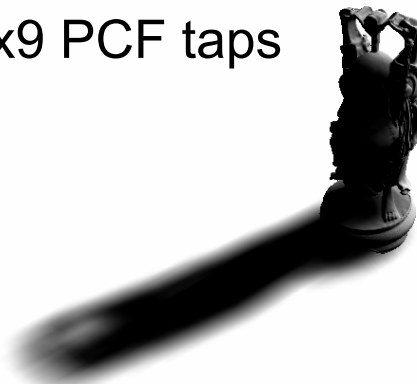
Cast penumbra with variable size

Shadows hardening on contact

1 PCF tap



9x9 PCF taps



PCSS





# Why Shadow Mapping for Soft Shadows?

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- ③ Most popular technique for shadows in games
  - ③ Purely image-based technique
  - ③ Works with any rasterizable geometry
- ③ Alternative techniques exist
  - ③ Silhouette based techniques such as smoothies and penumbra wedges
    - ③ Silhouette detection robustness issues
    - ③ Do not work with alpha-tested geometry



# Outline

## ③ Fixed-Size Penumbra

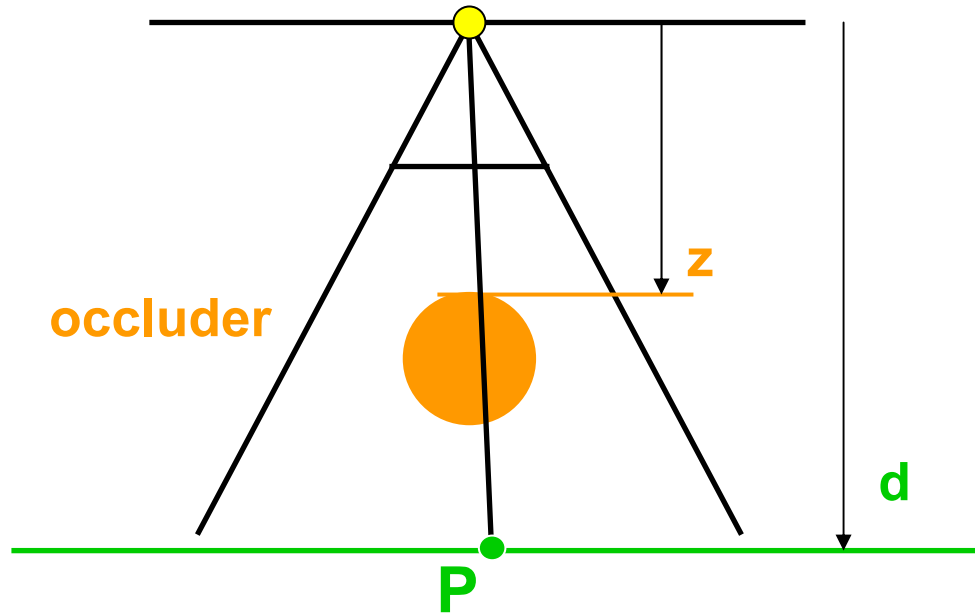
- ③ PCF (Percentage Closer Filtering)
- ③ VSM (Variance Shadow Maps)
- ③ CSM (Convolution Shadow Maps)
- ③ ESM (Exponential Shadow Maps)

## ③ Variable-Size Penumbra

- ③ PCSS (Percentage Closer Soft Shadows)
- ③ PCSS + VSM/CSM
- ③ Backprojection

# Shadow Mapping

- Shadow map stores distance  $z$  to the light
- $P$  is lit  $\iff (d < z)$





# Percentage Closer Filtering (PCF)

- ④ Sample the result of  $(d < z)$  around projected point
  - ④ Filter the binary results in a given kernel

0	1
1	0

- ④ Bilinear PCF
  - ④ NVIDIA and recent AMD GPUs implement 2x2 PCF in one fetch
  - ④ Using same sample locations and weights as for bilinear filtering



# Using Bilinear PCF with DX10

```
Texture2D<float> tDepthMap;
```

```
SamplerComparisonState ShadowSampler
```

```
{
```

```
    ComparisonFunc = LESS;
```

```
    Filter = COMPARISON_MIN_MAG_LINEAR_MIP_POINT;
```

```
};
```

```
// ...
```

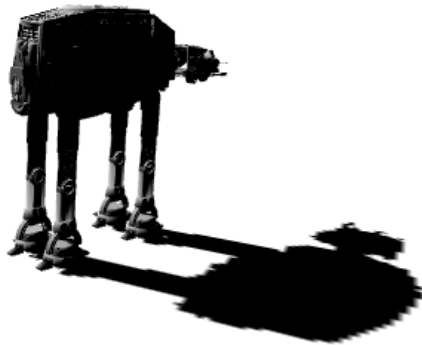
```
sum += tDepthMap.SampleCmpLevelZero(ShadowSampler,  
                                     uv + offset, z);
```





# PCF Filtering

- Increasing the number of PCF taps increases the softness of the shadows



1 tap



9x9 taps



17x17 taps



# Irregular PCF

- ⊕ PCF with large kernels requires many samples
- ⊕ Using irregular sampling
  - Trades banding for noise



regular sampling

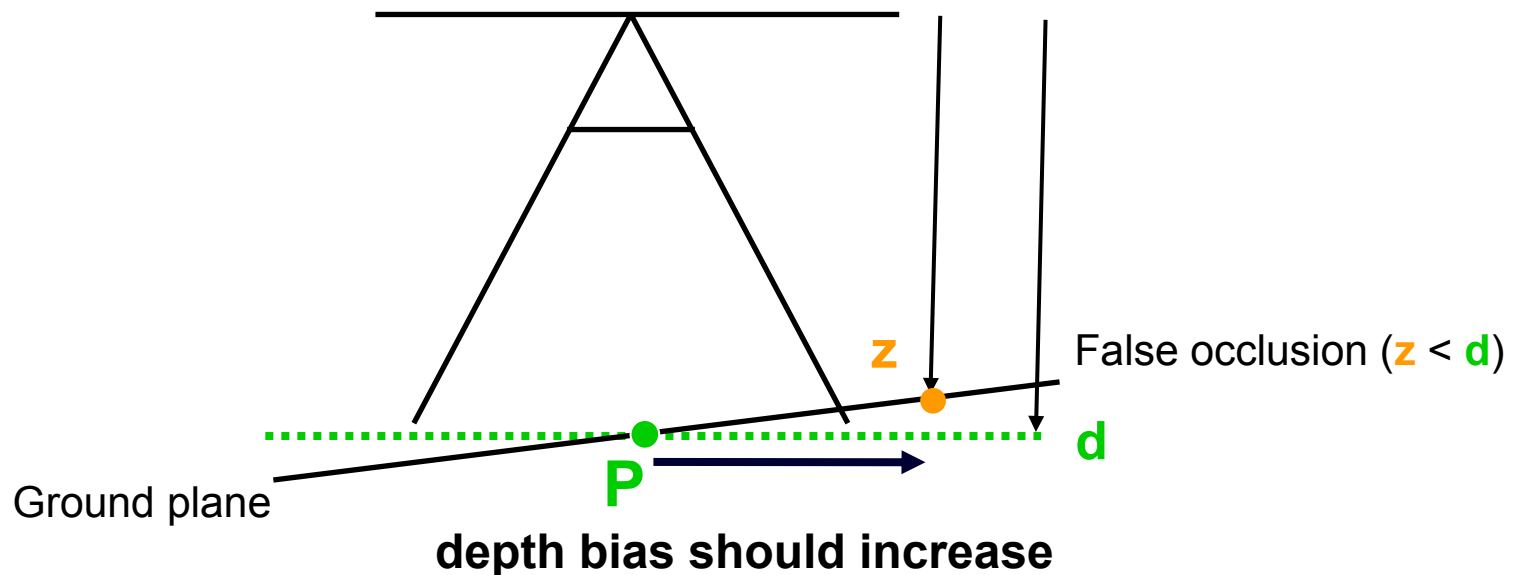


irregular sampling



# PCF Self-Shadowing Issue

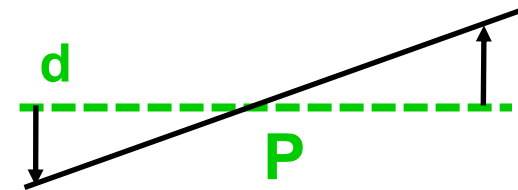
- Traditional depth bias: write  $(z + \text{bias})$  in shadow map
  - Bias = constant bias + slope-based bias
  - Issue: huge depth biases may be required for large PCF kernels



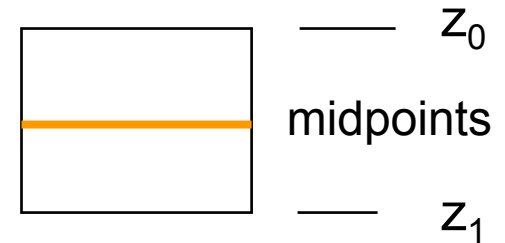


# PCF Self-Shadowing Solutions

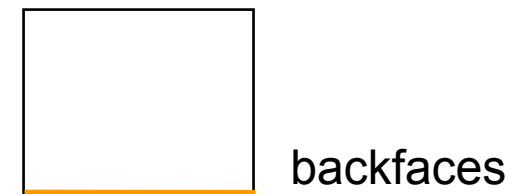
- Use depth gradient =  $\text{float2}(dz/du, dz/dv)$   
Make depth  $d$  follow tangent plane  
 $d = d_0 + \text{dot}(uv\_offset, \text{gradient})$   
[Schuler06] and [Isidoro06]



- Render midpoints into shadow map  
Midpoint  $z = (z_0 + z_1) / 2$   
Requires two rasterization passes
  - Depth peel two depth layersStill requires a depth bias for thin objects



- Render backfaces into shadow map  
Only works for closed objects  
Light bleeding for large PCF kernels

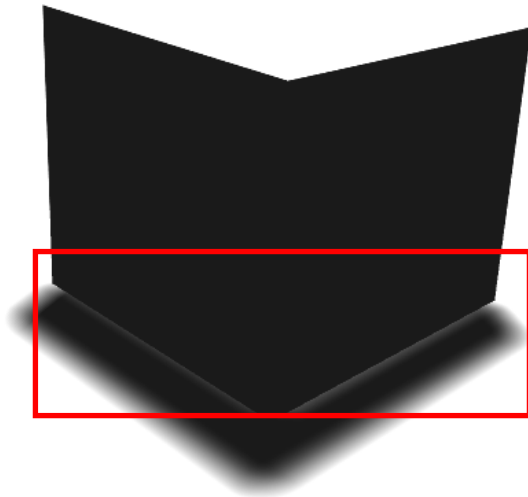




# Backfaces and Large PCF kernels

- ⦿ Rendering backfaces into shadow map generates light bleeding for large PCF kernels

Not due to FP precision or shadow map resolution  
But reverse of the surface acne issue





# Non-Linearity of PCF

- ④ PCF cannot be prefiltered as is
  - ④  $\text{Average}(d < z) \neq (\text{Average}(z) < d)$
  - ④ Filtering the depth buffer would smooth the heightfield of the shadow map
    - ④ Does not generate soft shadows
    - ④ May introduce artifacts
  
- ④ Solutions: Approximate shadow test by a linear function which can be prefiltered
  - ④ Goal: blurring the shadow map to generate realistic soft shadows



# Outline

## ④ Fixed-Size Penumbra

- ④ PCF (Percentage Closer Filtering)
- ④ VSM (Variance Shadow Maps)
- ④ CSM (Convolution Shadow Maps)
- ④ ESM (Exponential Shadow Maps)

## ④ Variable-Size Penumbra

- ④ PCSS (Percentage Closer Soft Shadows)
- ④ PCSS + VSM/CSM
- ④ Backprojection



# Variance Shadow Maps

- ⊕ Consider depth values in the filter kernel as a depth distribution [Donnelly06] [Lauritzen07]

- ⊕ Approximate the depth values in the kernel by a Gaussian distribution of mean  $\mu$  and variance  $\sigma^2$

$$\sigma^2 = E(z^2) - E(z)^2$$

$$\mu = E(z)$$

- ⊕ Probability of being lit

Using Chebyshev's inequality

$$P(d < z) \leq \max(\sigma^2 / (\sigma^2 + (d - \mu)^2), (d < \mu))$$

- ⊕  $E(X)$  = average of  $X$  in filter kernel

So the VSM ( $z, z^2$ ) can be prefiltered

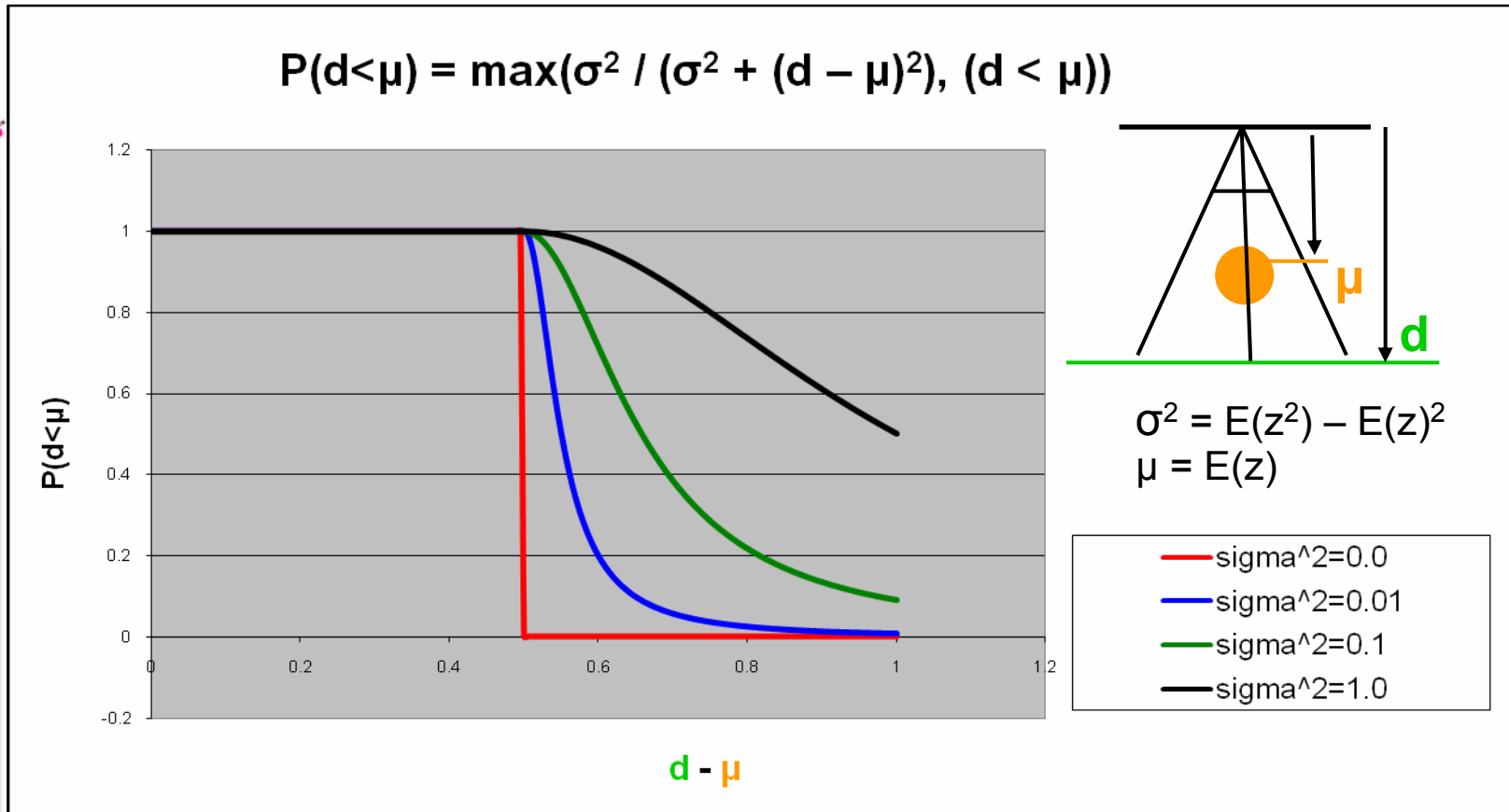
Can also be rendered using MSAA + resolve





# VSM Visibility Function

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- Self-shadowing can be handled simply by clamping  $\sigma^2$  to some small MinVariance parameter



# VSM Rendering

- ④ Blur  $z$  and  $z^2 \rightarrow E(z), E(z^2)$
- ④ Fetch  $E(z), E(z^2)$  with bilinear texture lookup  
Can also use trilinear and/or anisotropic filtering to reduce aliasing
- ④ Compute  $\mu = E(z)$  and  $\sigma^2 = E(z^2) - E(z)^2$   
If  $(\sigma^2 < \text{MinVariance}) \sigma^2 = \text{MinVariance}$
- ④ Light intensity approximation  
 $L = \max(\sigma^2 / (\sigma^2 + (d - \mu)^2), (d < \mu))$



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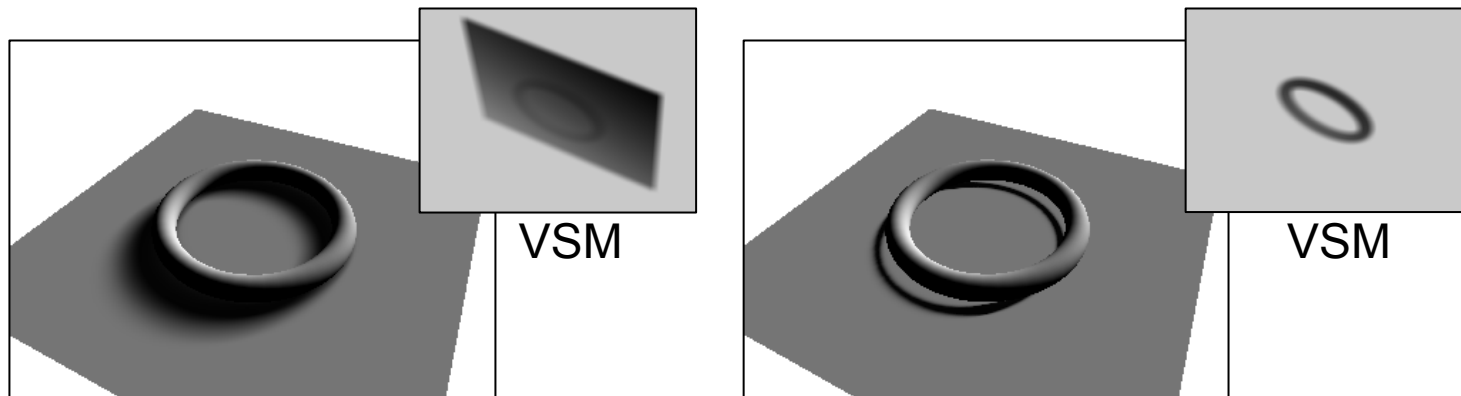
# Generating the VSM

- ④ Can use R32G32F\_FLOAT format
  - Filterable format on all DX10-class graphic hardware
  - Can also use FP16 [Donnelly06]
- ④ Render  $(z, z^2)$  to VSM texture
  - Where  $z$  is a linear distance
    - ④ Can use an orthographic projection
    - ④  $z = \text{mul}(\text{wPos}, \text{mObjectToLightOrtho}).z$
- ④ Clear VSM to  $(\text{MAX\_Z}, \text{MAX\_Z} * \text{MAX\_Z})$



# Rendering everything into the VSM shadow map

- Images with and without plane rendered in VSM

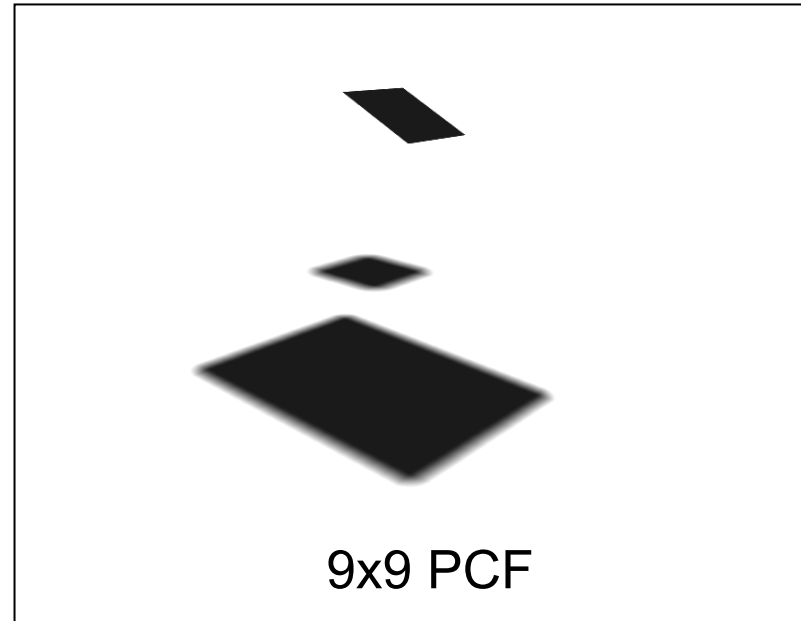
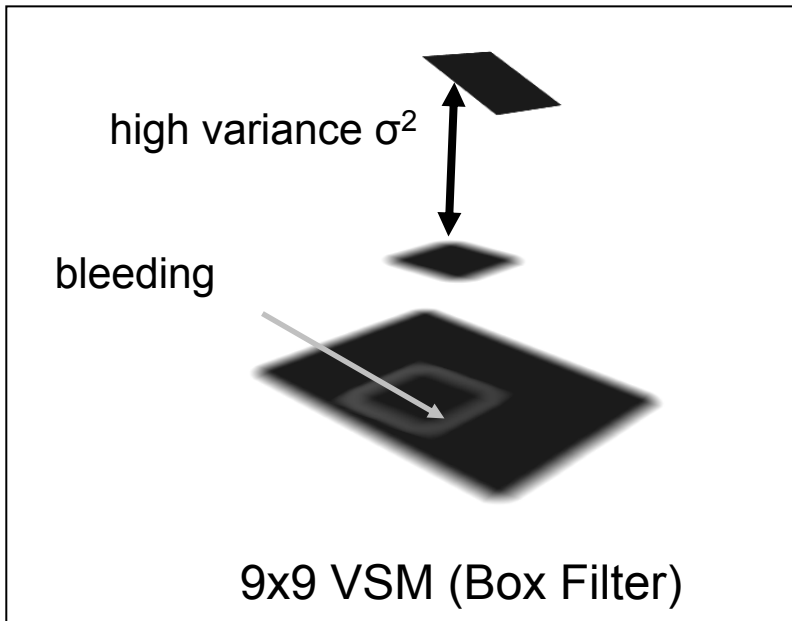


- If receivers are not in the VSM, will blur the object texels with  $z = \text{MAX\_Z}$  → missing occlusion
- CSMs and ESMs also have this limitation  
Shadows look bad when blurring shadow map without everything rendered into them



# VSM Light Bleeding

Two quads floating above a ground plane



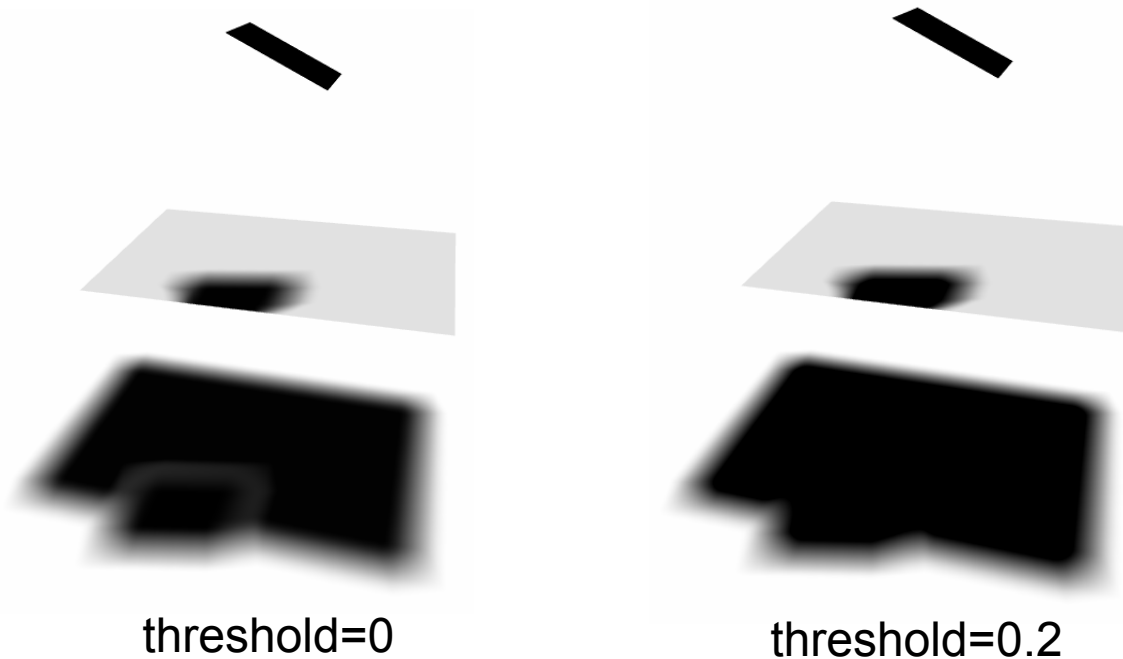
Large depth complexity in filter kernel

- $\sigma^2$  is large
- Upper bound  $\sigma^2 / (\sigma^2 + (d - \mu)^2) \rightarrow 1$



# VSM Light Bleeding

- Proposed in GPU Gems 3 [Lauritzen07]  
Use a threshold to remap shadow intensity

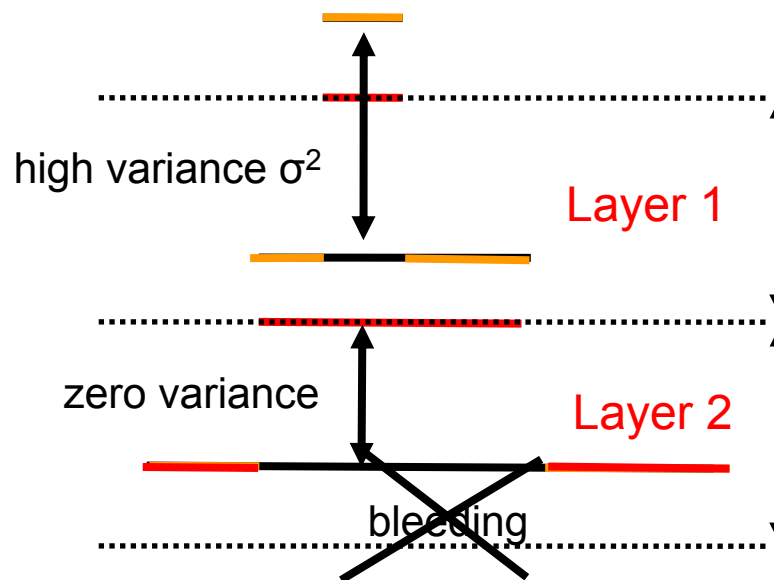




# Layered VSM

[Lauritzen08] I3D 2008

Partition shadow-map frustum into multiple depth ranges and clamp z to each range



VSM

Layered VSM



# Layered VSM

## ⊕ Compared to VSMS, LVSMs

Reduce or eliminate light bleeding artifacts

Do not require 32-bit float texture filtering

Can still be generated in one pass (using MRTs) and need only one texture sample per pixel

Variance Shadow Map



Layered Variance Shadow Map







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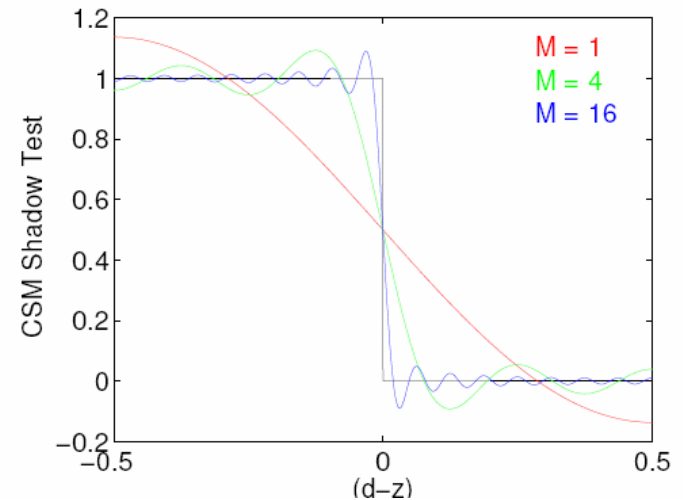
# Convolution Shadow Maps

- Approximate the step function (function of depth) by 1D Fourier expansion [Annen07]

$$f(d, z) \approx \frac{1}{2} + 2 \sum_{k=1}^M \frac{1}{c_k} \cos(c_k d) \sin(c_k z) - 2 \sum_{k=1}^M \frac{1}{c_k} \sin(c_k d) \cos(c_k z)$$

Function of  $d$   
Reconstruct when  
rendering shadows

Function of  $z$   
Write in shadow map  
and blur it



Excerpted from  
[Annen07]



# Generating a CSM

- ④ Assuming a linear depth  $z$  is available for every texel in the shadow map
  - $z$  needs to be normalized in  $[0,1]$  because the Fourier expansion is periodic
- ④ Can generate CSM with full screen shader pass
  - Rendering to R8G8B8A8\_UNORM MRTs
  - 4 MRTs ( $M=8$ ) is usually a good tradeoff
- ④ Background texels cleared to CSM(1.0f)



# CSM Rendering

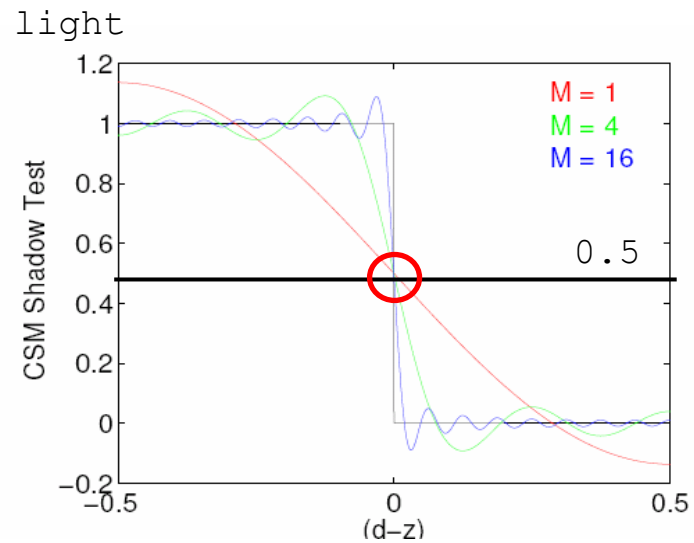
- Fetch  $\cos(c_k z)$  and  $\sin(c_k z)$  from the CSM textures and compute light intensity

$$f(d, z) \approx \frac{1}{2} + 2 \sum_{k=1}^M \frac{1}{c_k} \cos(c_k d) \sin(c_k z) - 2 \sum_{k=1}^M \frac{1}{c_k} \sin(c_k d) \cos(c_k z)$$

- Final light intensity

Scale by 2 otherwise  $f(d, z) == 0.5$  for  $(d == z)$

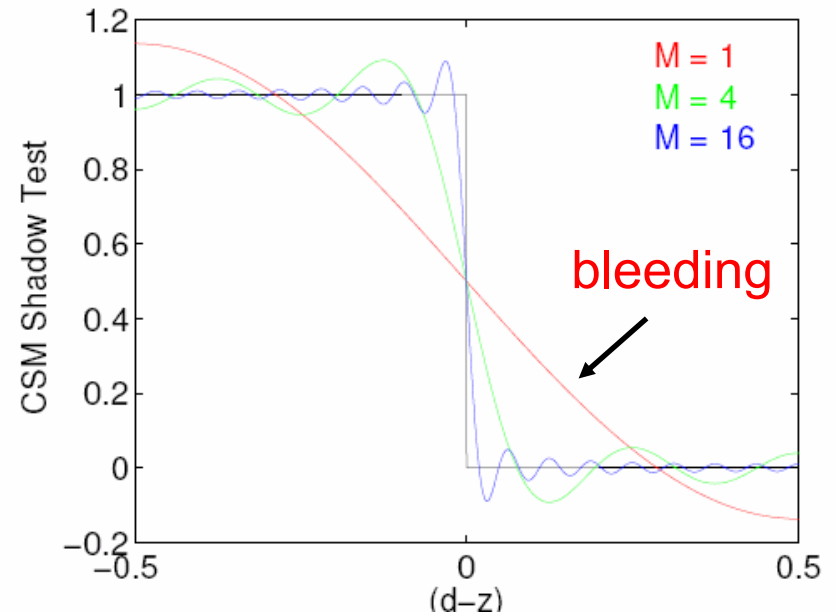
$L = \text{saturate}(2.0 * f(d, z))$





# CSM Light Bleeding

- ⊕ Limited bleeding may be seen as a feature (deep shadow look)
- ⊕ Light bleeding when should be in shadow ( $d > z$ ) but  $\text{light} > 0$





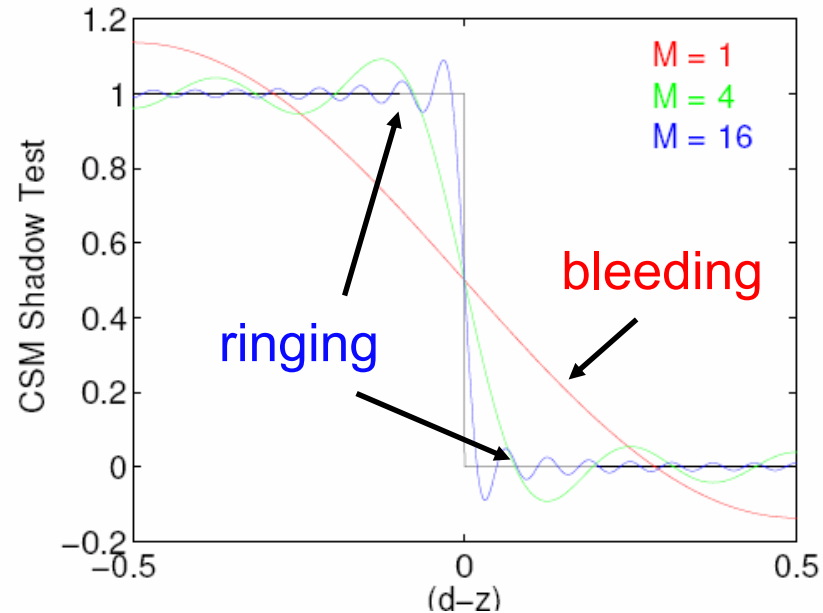
# CSM Ringing

- Gibbs phenomenon
  - High order Fourier terms generate ringing

- Ringings workaround
  - Attenuate terms based on their order (k)

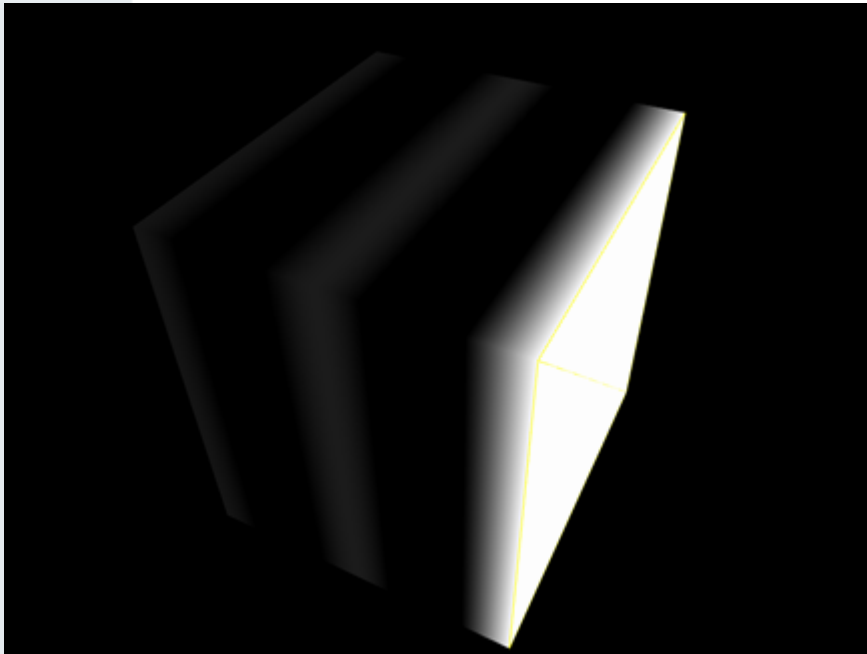
$$f(d, z) \approx \frac{1}{2} + 2 \sum_{k=1}^M \frac{1}{c_k} \cos(c_k d) \sin(c_k z) - 2 \sum_{k=1}^M \frac{1}{c_k} \sin(c_k d) \cos(c_k z)$$

\*  $\exp(-\alpha(k/M)^2)$

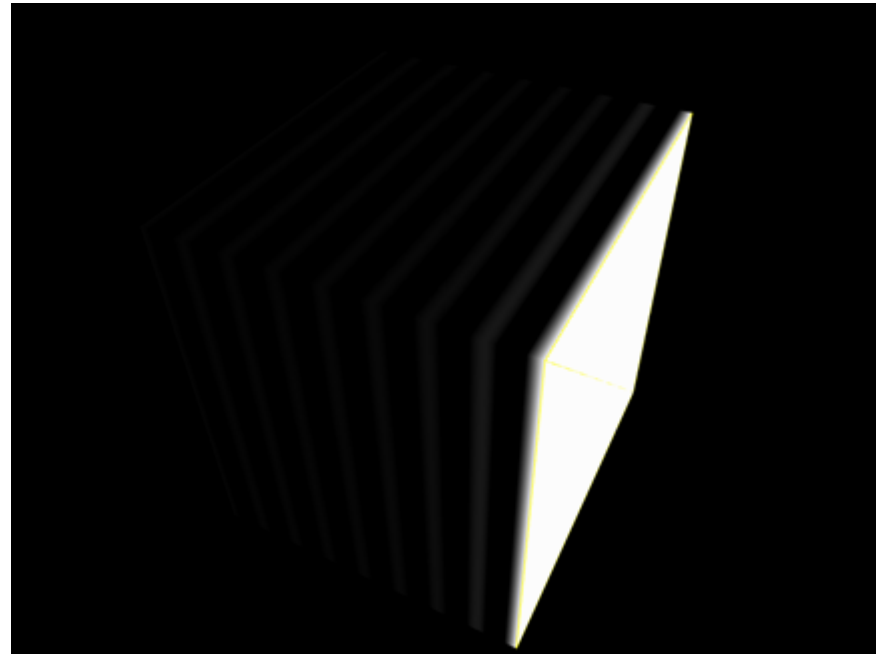


# CSM Ringing

- Light source facing inwards the cube



2 RGBA8

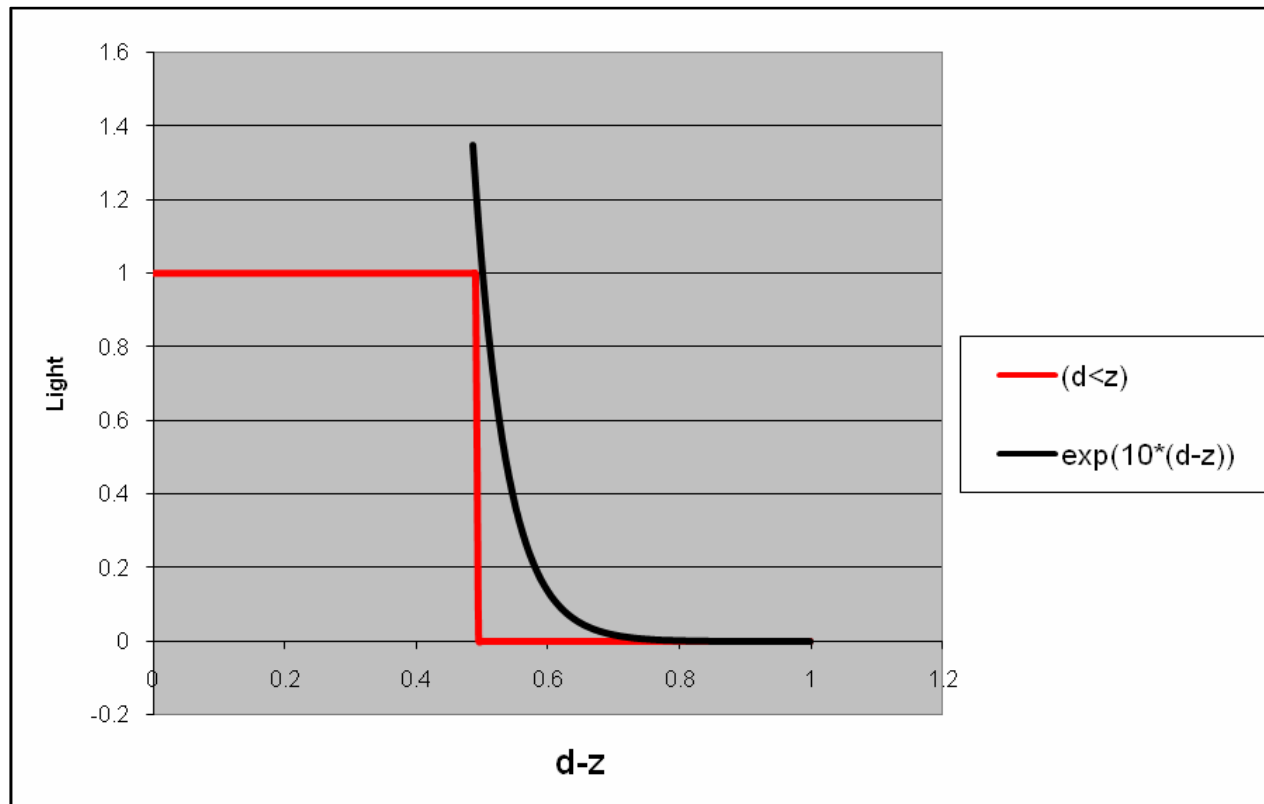


8 RGBA8



# Exponential Shadow Maps

- ④ [Salvi08] ShaderX<sup>6</sup>
- ④ Approximate step function ( $z-d > 0$ ) by  
 $\exp(k*(z-d)) = \exp(k*z) * \exp(-k*d)$







# ESM Parameter Tuning

$$L = \exp(k*z) * \exp(-k*d)$$



k=10

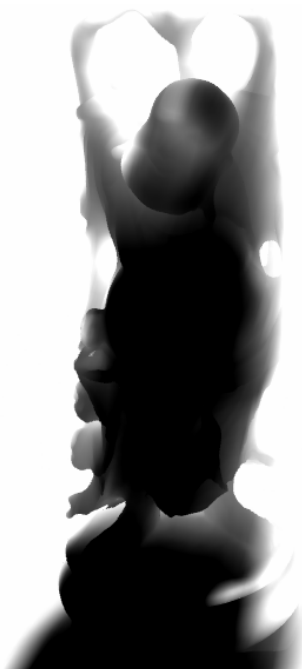


k=30



# Deep Shadow Look

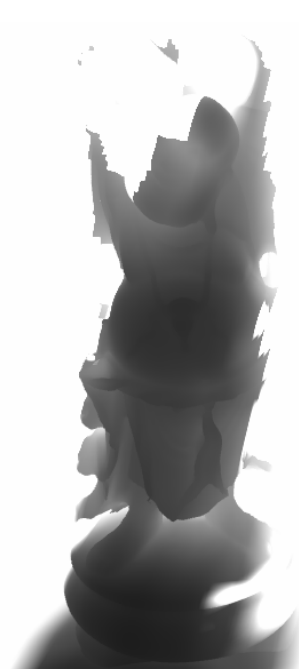
- ⊕ Shadows without any shading  
Soft shadow test gives translucent look



CSM



VSM



ESM



# VSM/CSM/ESM

## Parameters

VSM: MinVariance, BleedingReductionFactor

CSM: NumTextures, AbsorptionFactor

ESM: ScaleFactor

## Storage

VSM	CSM	ESM
R32G32	$N * (R8G8B8A8)$	R32

For CSM,  $N \geq 4$  is usually required to avoid bleeding

## The less storage, the faster the prefiltering



# VSM/CSM/ESM

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VSM – 87 fps



CSM – 38 fps

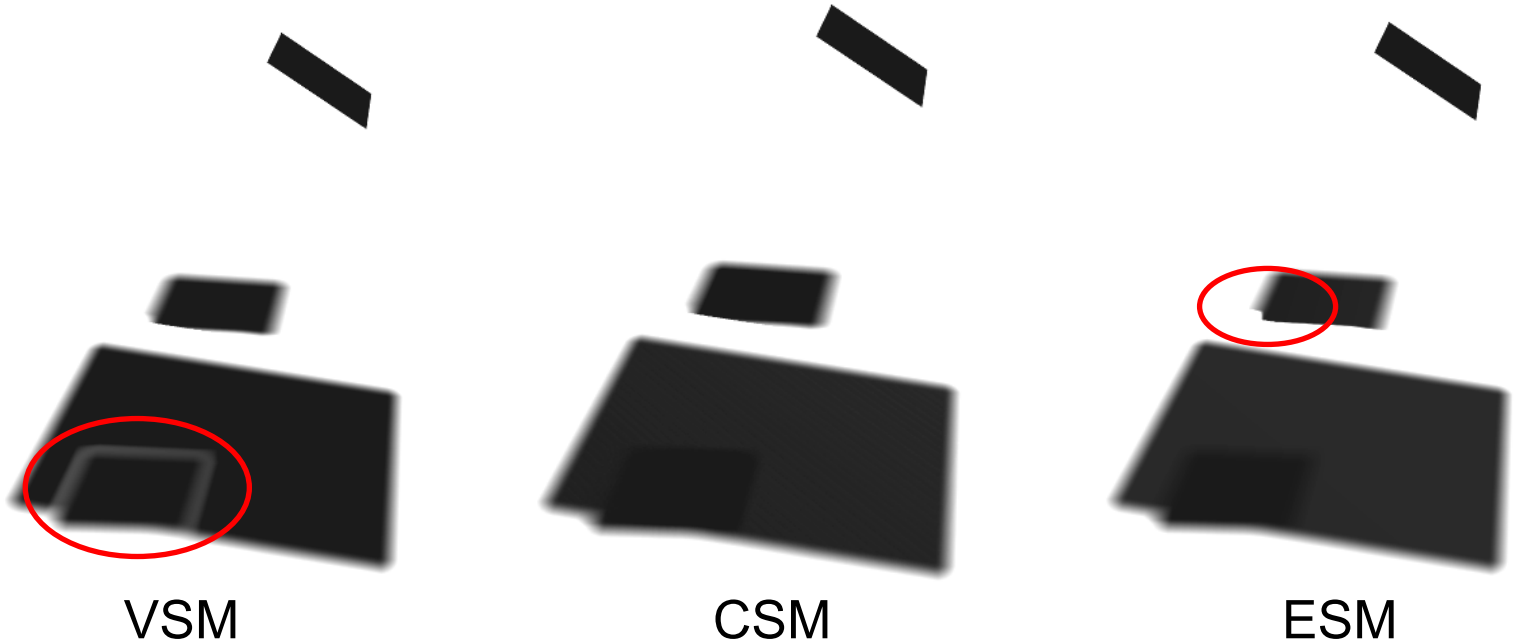


ESM – 93 fps



# VSM/CSM/ESM

- 2 quads floating above a ground plane



- The minor ESM artifacts can be handled with another parameter which overdarkens the shadows [Salvi08]



# Outline

## ④ Fixed-Size Penumbra

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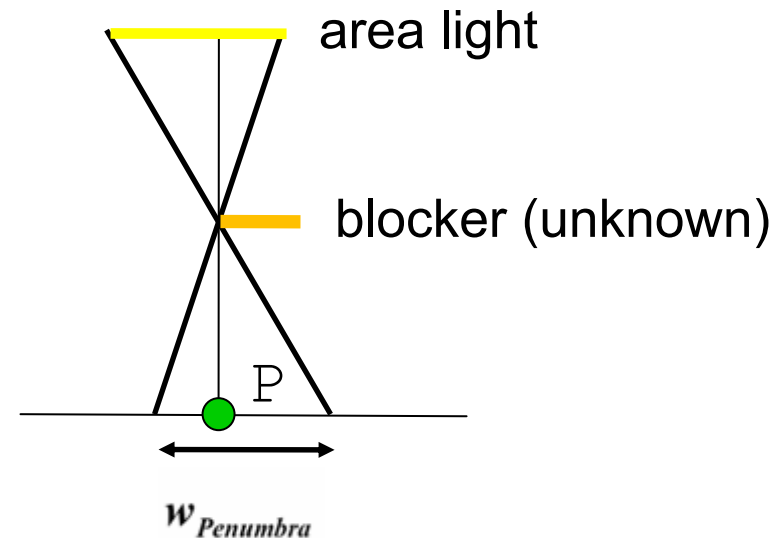
## ④ Variable-Size Penumbra

- ④ PCSS (Percentage Closer Soft Shadows)
- ④ PCSS + VSM/CSM
- ④ Backprojection



# Percentage Closer Soft Shadows

- ③ PCSS [Fernando05]
- ③ Assume a square light centered at the shadow map center
- ③ Assuming some parallel blocker to receiver  
Compute penumbra width using similar triangles



$$w_{Penumbra} = \frac{(d_{Receiver} - d_{Blocker}) \cdot w_{Light}}{d_{Blocker}}$$



# PCSS Overview

## ⌚ Step 1: Blocker Search

- ⌚ Sample the depth buffer using point sampling
- ⌚ Average all blockers with  $(\text{depth} + \text{bias} < \text{receiver})$  in search region / kernel
- ⌚ Early out if no blocker found

## ⌚ Step 2: Filtering

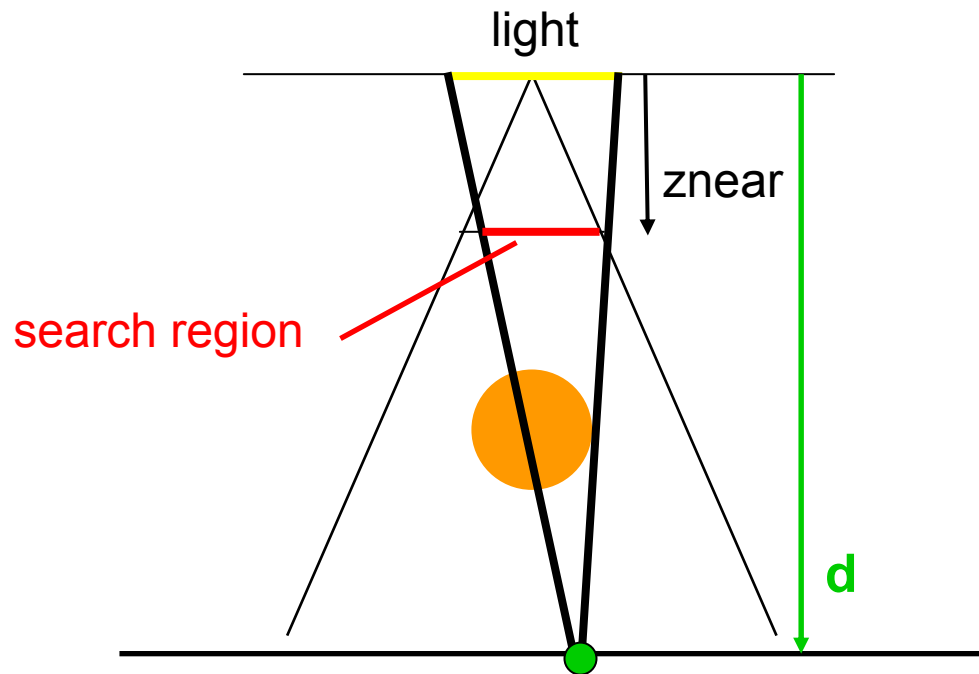
- ⌚ Use filter radius from step 1
- ⌚ Clamp filter width to be  $\geq \text{MinRadius}$  for antialiasing
- ⌚ Filter the shadow map with PCF or VSM/CSM/ESM





# Search Region

- Where to find blockers?
- Conservative search radius using similar triangles

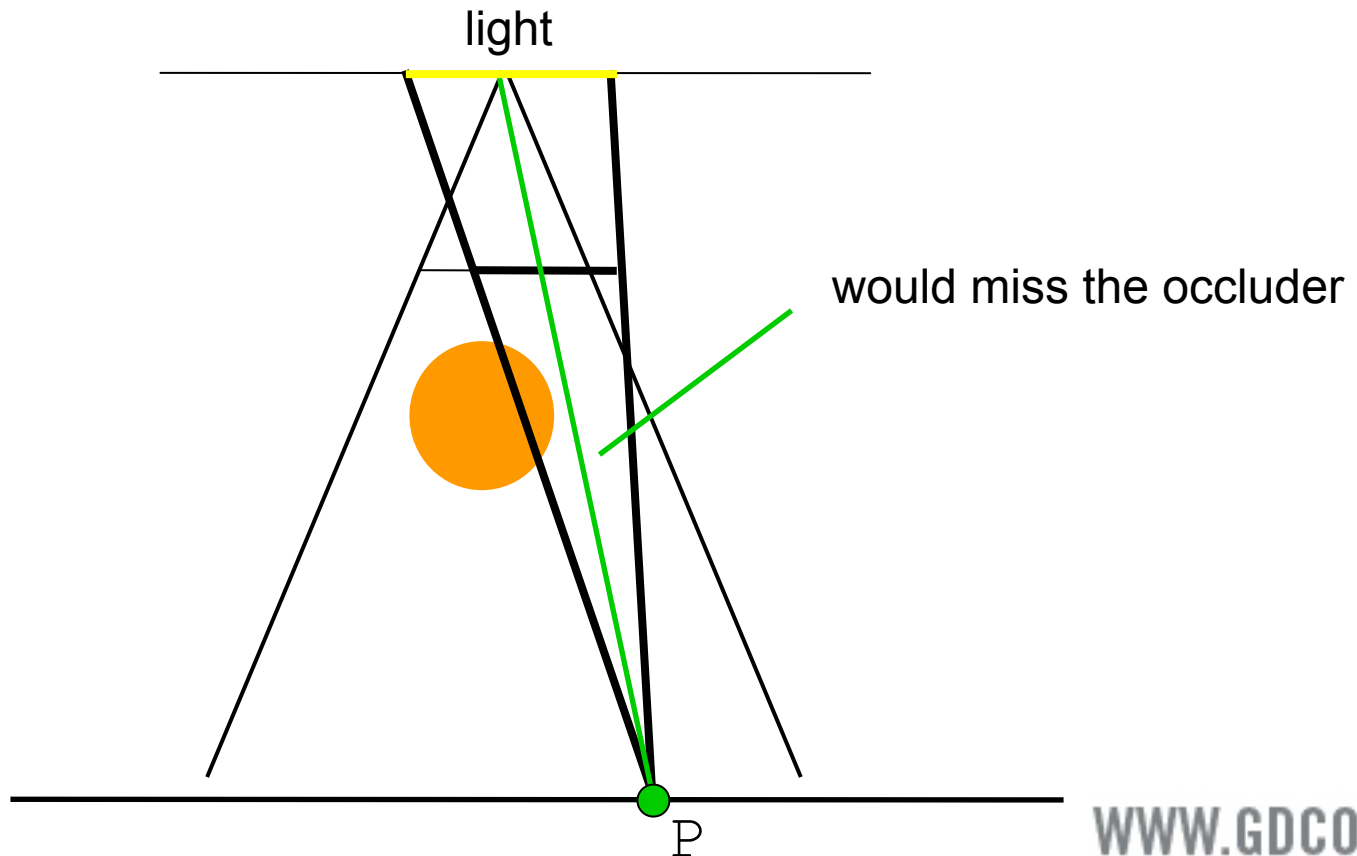


$$\text{LightRadius} / d = \text{SearchRadius} / (d - \text{znear})$$



# Blocker Search

- Why not doing just one sample?





# Blocker Search

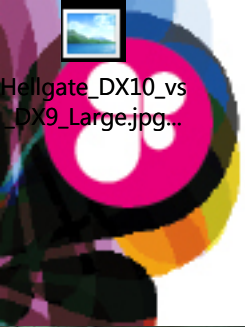
- ⊕ The more samples in the blocker search, the less noisy artifacts in the soft shadows  
In practice, 4x4 or 5x5 samples is sufficient



Blocker Search with 3x3 taps



Blocker Search with 5x5 taps



# PCSS In Hellgate: London



PCSS



PCF

16 POINT taps for the blocker search  
16 PCF taps for the PCF filtering



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# PCSS + VSM/CSM/ESM



- ④ Compute filter size using blocker search
- ④ Solutions to prefilter the shadow map to support fast blurs of variable size
  - ④ Summed Area Tables (SAT)
    - ④ Fast Box Filters
  - ④ Mipmapping
    - ④  $\text{Level}[i] = \text{Blur}(\text{Level}[i-1])$



# Summed Area Tables

- ⊗ For a 1D texture, each texel  $S[n]$  of the SAT is  
$$S[n] = \text{Sum}(T[i], i=0..n)$$
- ⊗ Average in interval  $(i,j]$  is  
$$(S[j]-S[i]) / (j-i)$$
- ⊗ Also works in 2D  
With more 4 fetches instead of 2
- ⊗ Fast algorithm for building SATs on GPUs [Hensley05]  
Recursive doubling  
For  $N \times N$  shadow map,  $\log(N)$  passes



# Summed Area Shadow Maps

## ⌚ Using UINT32

- ⌚ Advantages: more bits available than FP32, and support arbitrary shadow map resolution
- ⌚ Drawback: need to implement bilinear filtering in shader
- ⌚ See GPU Gems 3 chapter for details [Lauritzen07]

## ⌚ Best fitting formats for DX10 GPUs

- ⌚ For SAVSM - R32G32\_UINT
- ⌚ For SACSM - multiple R32G32B32A32\_FLOATs
- ⌚ For ESM (storing  $\exp(k*z)$ ) - R32\_UINT





# SAVSM vs MIPCSM

- ⊗ Disadvantage of SAT: can only do box filter
- ⊗ Mipmap with recursive blurs looks more like a Gaussian



SACSM



MIPCSM



# SATs with variable size kernel

- ⊗ Rounding the filter radius generates banding

```
float2 moments = FilterMomentsBilinear(uv, round(filterRadius));
```

- ⊗ Better quality: trilinear filtering

```
float2 moments0 = FilterMomentsBilinear(uv, floor(filterRadius));
```

```
float2 moments1 = FilterMomentsBilinear(uv, ceil(filterRadius));
```

```
float2 moments = lerp(moments0, moments1, frac(filterRadius));
```



# SATs for VSM/CSM/ESM

- ⊕ Cost of building a SAT (on G80 Ultra)

  - SAVSM

    - ⊕ 0.9 ms for  $512^2$ , 3.3 ms for  $1024^2$

  - SACSM with 16 FLOATs per texel

    - ⊕ 6.3 ms for  $512^2$  and 27.5 ms for  $1024^2$

  - SAESM

    - ⊕ 0.6 ms for  $512^2$ , 2.2 ms for  $1024^2$

- ⊕ CSM is not well suited for SATs

  - Mipmaps are a better fit for them



# MIPCSM

- ④ Build a mipmap pyramid on top of CSM
  - Implemented as texture array with mip levels
  - How to generate the mipchain
    - ④ Default 2x2 mipchain generates aliasing
    - ④ Use a larger box filter (at least 7x7) to generate each LOD, double spacing every level
    - ④ Lookup CSM using trilinear filtering
- ④ Advantages of MIPCSM over SACSM
  - ④ Can use RGBA8 instead of RGBA32F
  - ④ Much faster to build than SACSM



# MIPCSM

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- ⊕ To reduce issues to discretized blurs, do a small blur pre-pass over the CSM



without pre-blur



with 5x5 pre-blur



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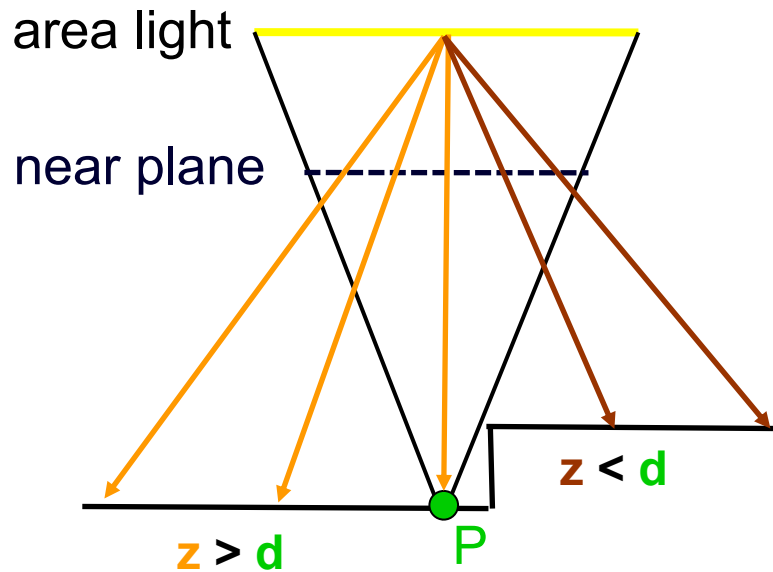
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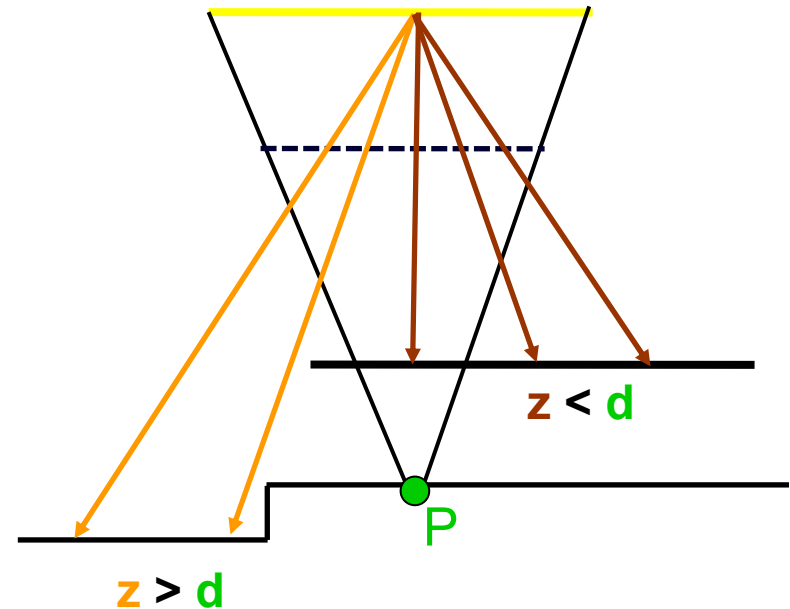
- ④ PCSS (Percentage Closer Soft Shadows)
- ④ PCSS + VSM/CSM
- ④ **Backprojection**



# Why PCF is wrong for soft shadows



PCF: Shadow(P) > 0  
Ground Truth: Shadow(P) == 0

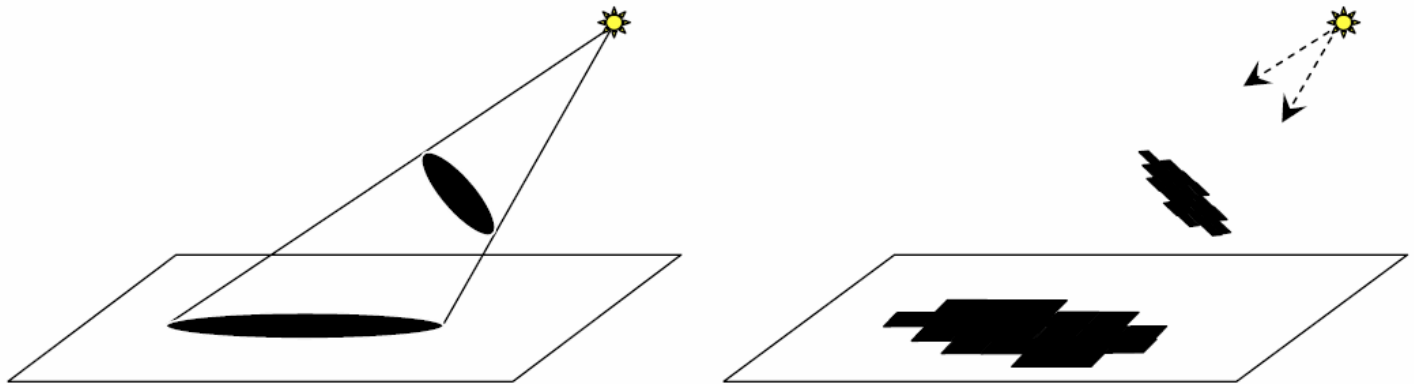


PCF: Shadow(P) < 1  
Ground Truth: Shadow(P) == 1



# Unprojecting the shadow map

- ③ Unproject shadow map texels into world space [Atty05] [Guennebaud06]
- ③ Discretized approximation of the scene geometry

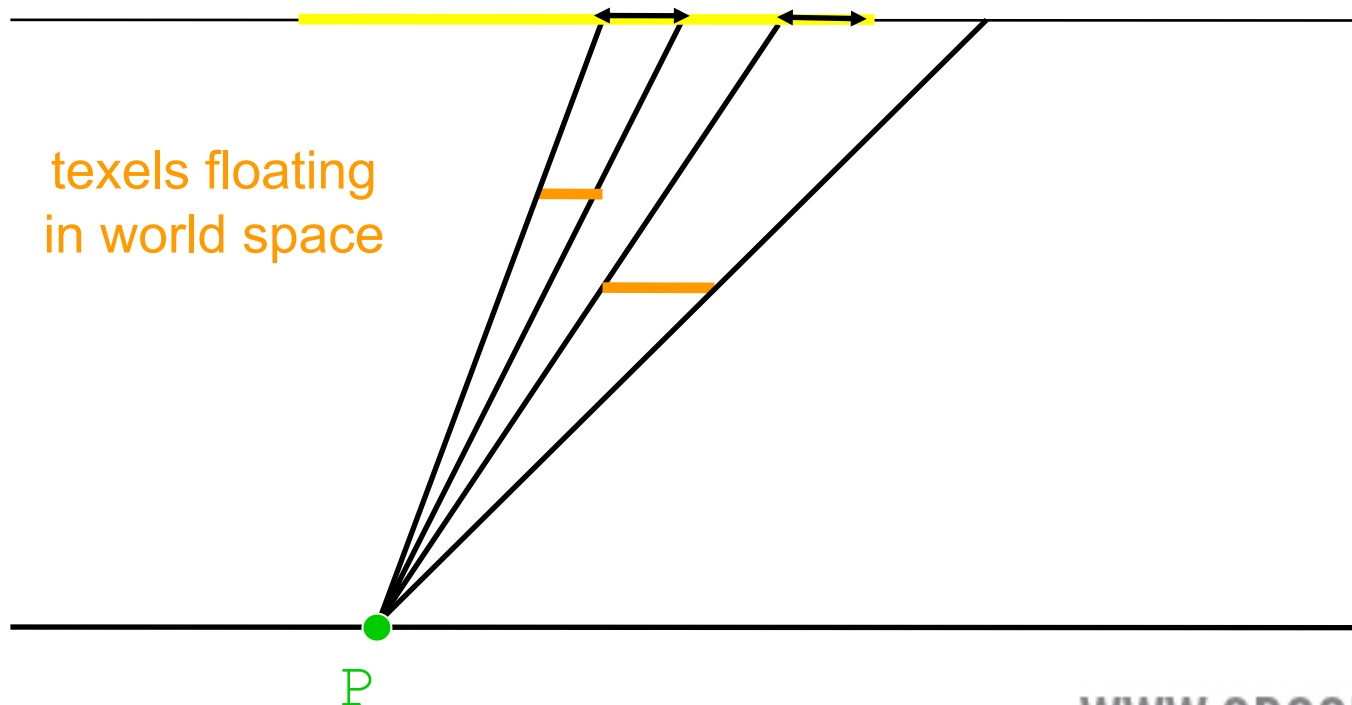






# Backprojecting texels

- ⊕ Backproject texels onto light source
- ⊕ Clamp to light borders and accumulate area





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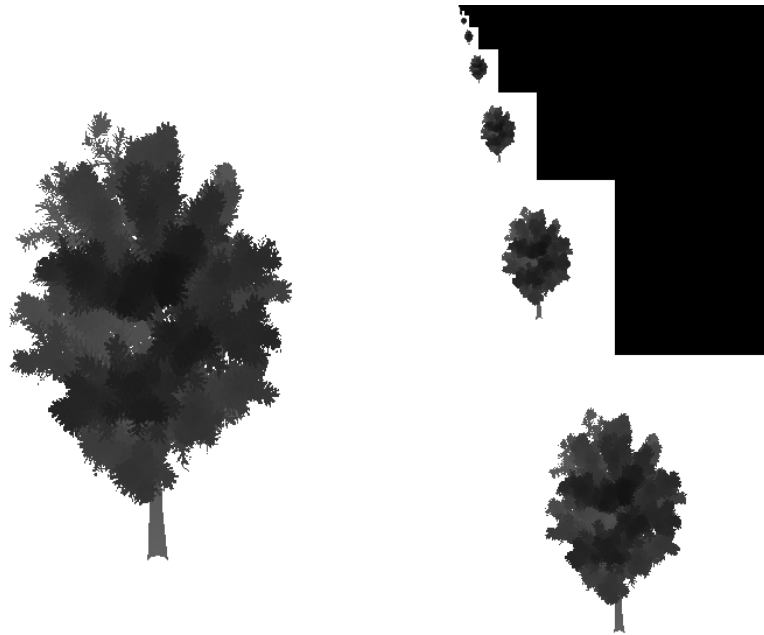
# Min-Max Mipmap Shadow Map

- ⌚ Can represent shadow-map data in hierarchical fashion [Dmitriev07]
  - Hierarchical traversal allows for efficient pruning of subtrees
- ⌚ Build a hierarchical shadow map first
  - ⌚ R32G32\_FLOAT mipmap
  - ⌚ Each level stores min and max z of 2x2 subpixels
- ⌚ Links
  - [SDK sample](#)
  - [GDC 2007 talk](#)



# Flattening the mipmap

- ④ Generate mipmap and flattened 2D texture
  - ④ Load() from texture without mipmaps is faster because it guarantees that all pixels access the same mipmap level.



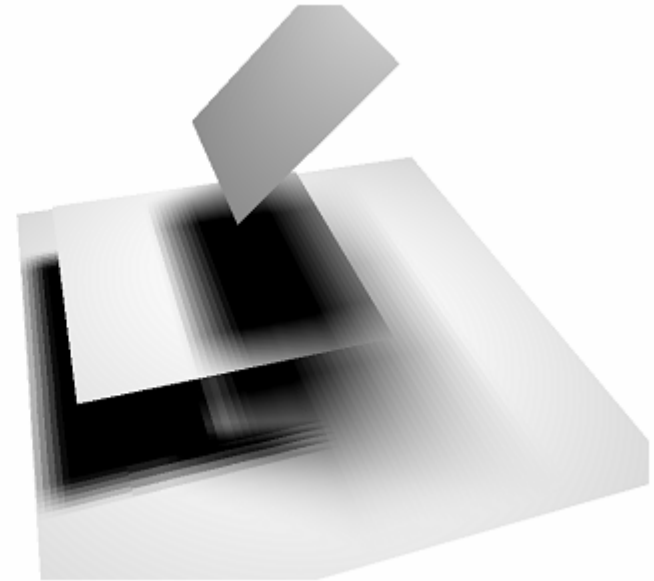
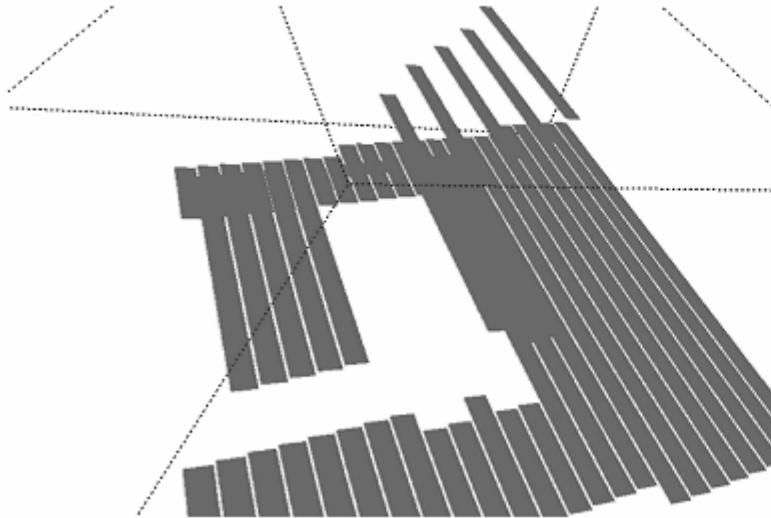
Flattened mipmap



# Light Bleeding

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- ⊕ Light bleeding is caused by gaps between micropatches





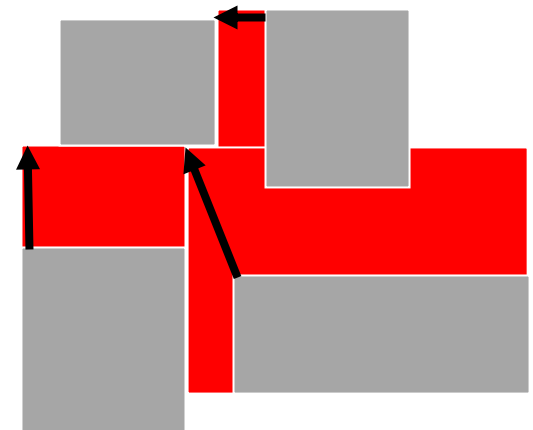
# Gap Filling

- Extend backprojected quads on the sides to touch their left and top neighbors and the diagonal

Similar to [Guennebaud06]



backprojected texels



filling the gaps

# PCSS with PCF

**Frame Rate: 120 fps (8.3 ms)**

**Shadow Map Generation: 1.4 ms**

Blocker Search: 5x5 taps

PCF: 5x5 taps



Image: 1600x1200  
Shadow Map: 1024<sup>2</sup>  
Triangle Count: 211k  
GeForce 8800 GT

# PCSS with PCF

**Frame Rate: 96 fps (10.4 ms)**  
**Shadow Map Generation: 1.4 ms**

Blocker Search: 5x5 taps  
PCF: 9x9 taps

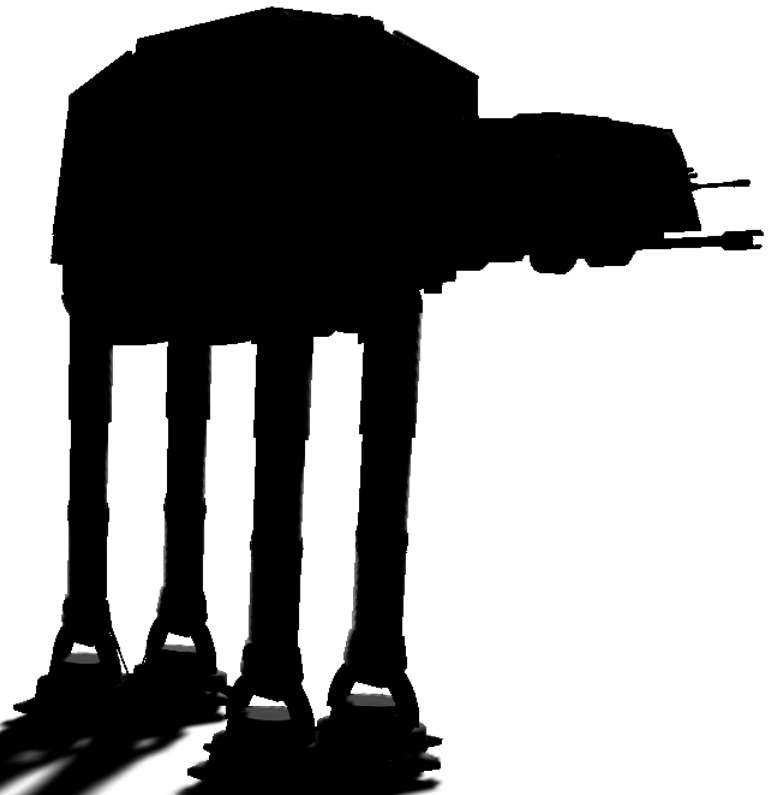


Image: 1600x1200  
Shadow Map: 1024<sup>2</sup>  
Triangle Count: 211k  
GeForce 8800 GT

# PCSS with PCF

**Frame Rate: 54 fps (18.5 ms)**  
**Shadow Map Generation: 1.4 ms**

Blocker Search: 5x5 taps  
PCF: 17x17 taps

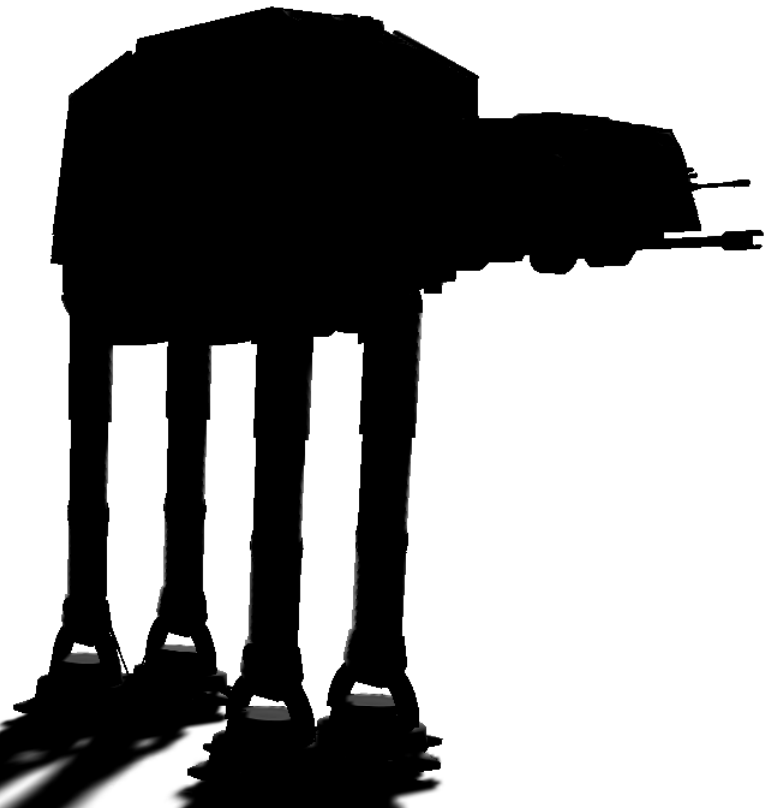


Image: 1600x1200  
Shadow Map: 1024<sup>2</sup>  
Triangle Count: 211k  
GeForce 8800 GT



# PCSS + SAVSM

**Frame Rate: 68 fps (14.7 ms)**  
**SAVSM: 6.5 ms total (44%)**

Blocker Search: 5x5 taps  
UINT32 SAVSM

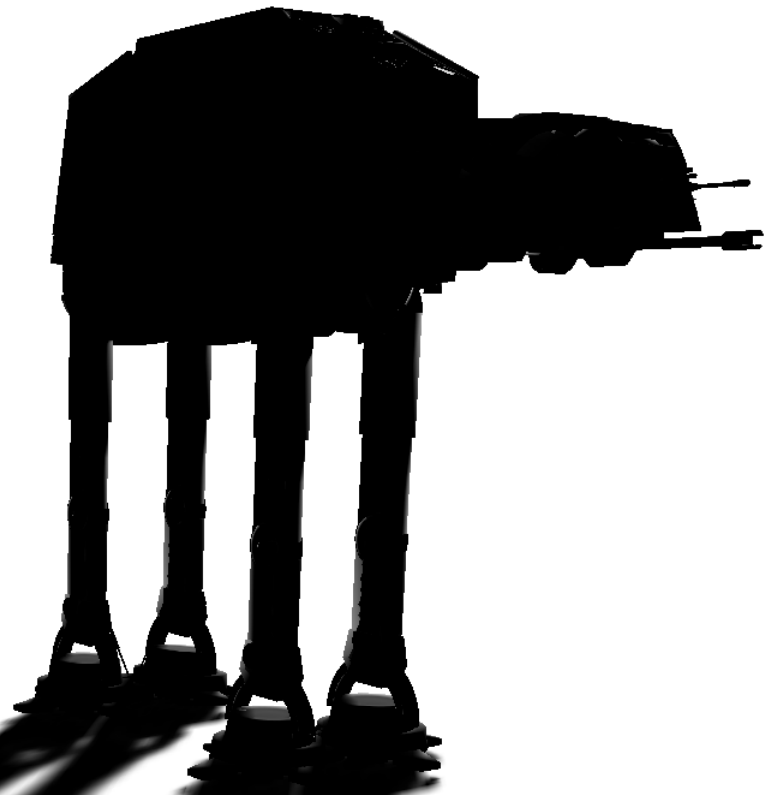


Image: 1600x1200  
Shadow Map: 1024^2  
Triangle Count: 211k  
GeForce 8800 GT

# PCSS + MIPCSM

**Frame Rate: 57 fps (17.5 ms)**  
**MIPCSM: 9.4 ms total (54%)**

Blocker Search: 5x5 taps  
MIPCSM: 4 RGBA8

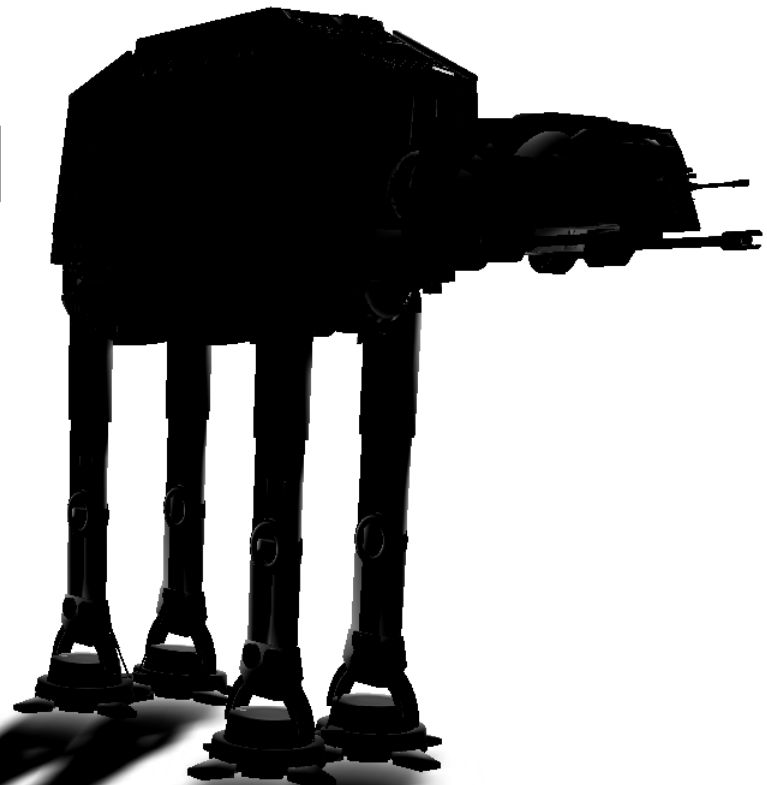


Image: 1600x1200  
Shadow Map: 1024^2  
Triangle Count: 211k  
GeForce 8800 GT

# Hierarchical Backprojection

Frame Rate: 11 fps (91 ms)  
Min-Max Mipmap: 2.6 ms (3%)

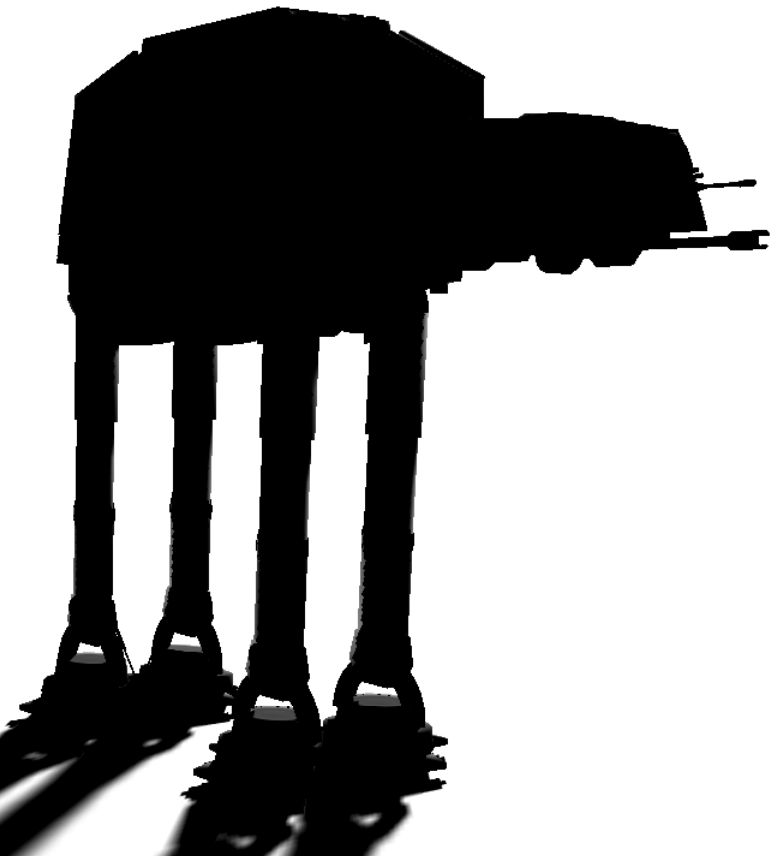
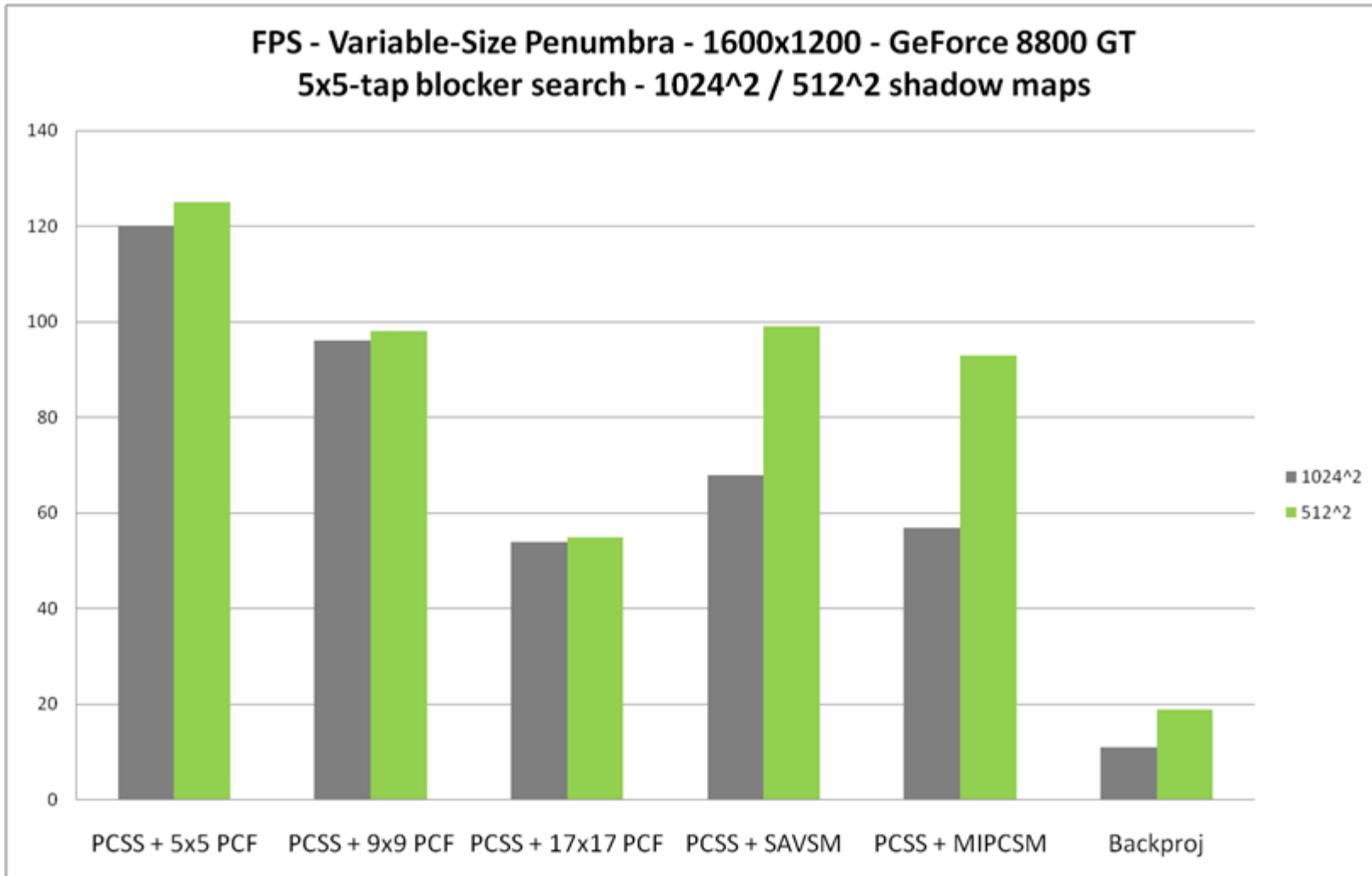


Image: 1600x1200  
Shadow Map: 1024<sup>2</sup>  
Triangle Count: 211k  
GeForce 8800 GT



# Performance Comparison

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# Conclusions

- ⊕ VSM/CSM/ESM all have some sort of light bleeding

There are ways to control it

- ⊕ VSM: threshold and/or layered VSM [Lauritzen08]
- ⊕ CSM: more coefficients [Annen07]
- ⊕ ESM: scale factor [Salvi08]

- ⊕ Percentage Closer Soft Shadows (PCSS) is good stuff

Used in shipping game (Hellgate: London)

See our latest [PCSS whitepaper](#) (February 2008)

Two step process:

- ⊕ 1. compute filter size using point sampling
- ⊕ 2. filter the shadow map using PCF



# Conclusions

- ④ Can swap PCF filtering with other filtering methods
  - PCSS + SAVSM works great
    - ④ See GPU Gems 3 Chapter for SAVSM (including source code)
    - ④ Can reduce light bleeding using layered VSMs
- ④ Ideas for accelerating the blocker search
  - Min-max mipmap hierarchical traversal
  - Summed Area Table in the case where the shadow receivers are not in the PCSS shadow map
- ④ Shadow map generation recommendation
  - Use a separate stream for the depth pre-pass with the minimum set of vertex attributes



# Acknowledgments



## ⌚ NVIDIA

Kevin Myers

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⌚ Marco Salvi (ESM)

⌚ Andrew Lauritzen (VSM)



# Resources

## ⊕ Implementation details

### PCSS

- ⊕ [PCSS integration guide](#) (February 2008)
- ⊕ DirectX 10 sample to be released soon

### Hierarchical backprojection

- ⊕ [SoftShadows](#) sample in our SDK10

### SAVSM

- ⊕ GPU Gems 3 chapter

## ⊕ Models

AT-AT: Brad Blackburn / [www.scifi3d.com](http://www.scifi3d.com)

Trees: Generated using [Dryad](#)

Buddha: Stanford Mesh Repository





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